Everything you wanted to know about TWA – VMG – VMC

by J.J. JACQ

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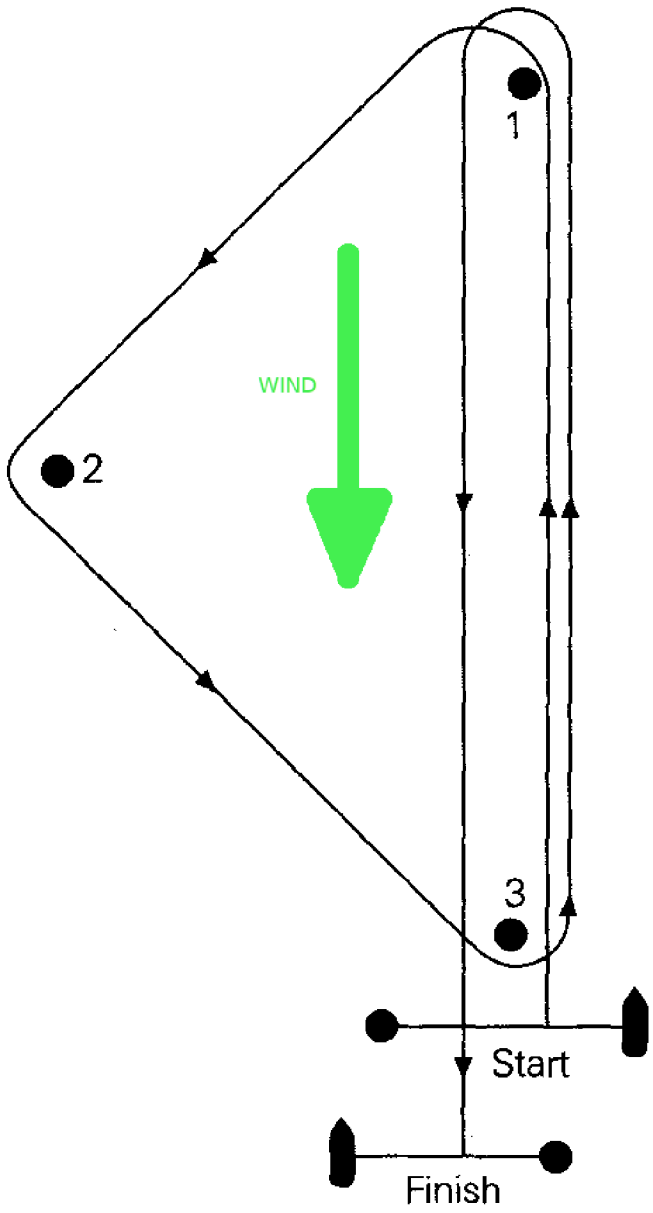
1. Basis

Traditionally, boat races are carried out on a course around three buoys which are set such that the leg from starting line to buoy No 1 is directly again the wind, from No 1 to No 2, boats have the wind at 135 degrees, and the same from No 2 to No 3.

Typically boats sail from start, to #1 then #2 then #3, then again to # 1 and finally to the finish.



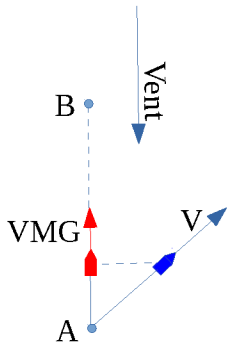
Sail boats (LS Xmaswhale excepted) are unable to sail directly against the wind and there are two legs where they face the wind. They have to tack back and forth and travel in zigzag fashion. They use sails like the staysail, solent, jib, or Genoa depending on the wind strength. Sails like Code 0, reacher, gennaker or spinnaker are no good for this type of sailing. The angle at which the boats sail is called True Wind Angle (TWA). The efficiency of the sails is plotted as a function of wind speed and TWA and with those two figures one obtains the boat speed that the sail can achieve. The plot is usually shown in polar form and is called a polar diagram.

The sailors' problem is to choose the angle which will make them reach the #1 buoy in the shortest possible time. For this they must pick the angle that maximises the Velocity Made Good (VMG).

2. VMG – What is that?

Picture two boats, one a sailboat, the other a motor boat. Both wish to go from A to B directly against the wind. The sail boat is sailing with the wind at an angle TWA and its speed is V obtained from the polar diagram. The motor boat is proceeding directly to B but with such a speed that an observer on board can see the sail boat exactly abeam on its starboard side. The speed of the motor boat is the VMG (velocity made good) of the sailboat.

When the motor boat is midway between A and B, the sail boat tacks and comes back still with the same TWA but on the opposite side. Both boats arrive together at B



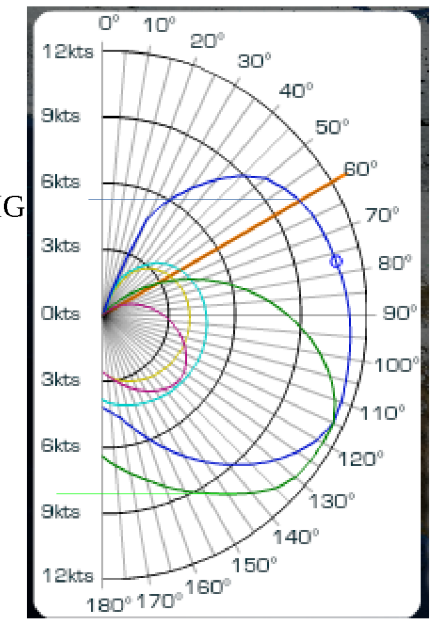
For instance, if TWA is exactly 60 degrees, the VMG will be exactly half of V. If TWA is 90 degrees, the VMG will be 0. If TWA is greater than 90, the VMG is negative as the boats move away from B rather than near it.

If B is not exactly on the wind axis, but is not too far from it, experience shows that tacking back and forth at the TWA that maximises VMG is still the best solution. The tacks are no longer symmetrical, there is a short one and a long one, but the TWA is still the same on each tack and this still minimises the time to sail from A to B.

RG does not display the polar diagram in a conventional manner. They show TWA = 0 on the right hand side. The conventional manner is to show 0 at the top the same as 0 heading on a compass. Here is a more conventional display of a polar diagram.

VMG

VMGmin



The red line shows the boat heading in relation to the wind (60° in this instance). The boat speed is 10 knots. VMG is read on the left axis and is 5 knots in this instance. It is obvious that if the boat reduces its TWA, the boat speed will decrease but at first the VMG will increase. It will be maximum when the TWA has been reduced to 45°, at this stage the boat speed will be 9 knots instead of 10, but the VMG will be 6.36 knots instead of 5. If the TWA is reduced further, the boat speed will continue to decrease but then so will the VMG.

If we wish to travel with the wind instead of against it, the same principle applies, except that if one sails with a TWA = 180°, the boat will still move. However on modern sailboats, a TWA around 1140 ~ 150° is better. On our polar diagram, a TWA of 143° shows the boat moving with a VMG of 8 knots, with a TWA of 180° the VMG is 6 knots only.

For those of you not frightened by mathematical formulas, here is the formula to obtain the VMG once the boat speed and TWA are known:

$$VMG = V \times \cos(TWA)$$

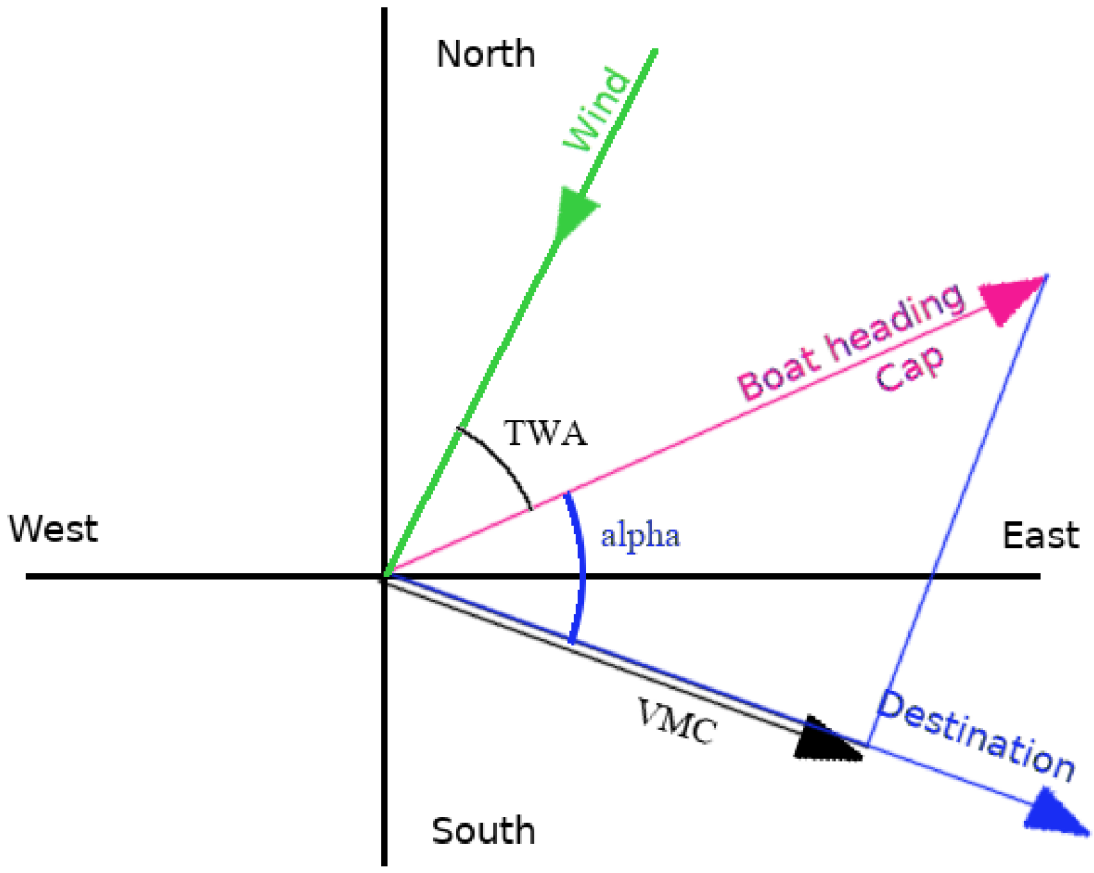
3. VMC – What's this for?

VMG is very useful when the required heading is close to the wind axis. But if the destination is in another direction we can use a similar analysis.

We call VMC the speed of the boat projected on the axis of the destination, VMC = Velocity Made good to Course.

Just as in a regatta we seek to maximise the VMG, in a long distance race one may seek to maximise the VMC. To do so, one must first determine the direction of the destination. A handy little tool for this purpose can be found at: https://madinstro.net/sundry/navigation/orthodrome.html

Next, given the boat heading, we get "alpha", the angle between the boat heading and the destination, and as before we also get TWA, the angle between the wind and the boat heading. TWA and the wind speed give us the boat speed (and the best sail to use for this heading) , and alpha gives us the VMC.



VMC is derived with a formula similar to that of the VMG, but with alpha instead of TWA as angle:

$$VMC = V \times \cos(alpha))$$

There is however a major difference between VMC and VMG. When we sail against or with the wind, we can tack or gybe back and forth each tack having the same TWA angle. With the VMC we can't do that. There is a heading which maximises the VMC but there is generally no alternate tack heading.

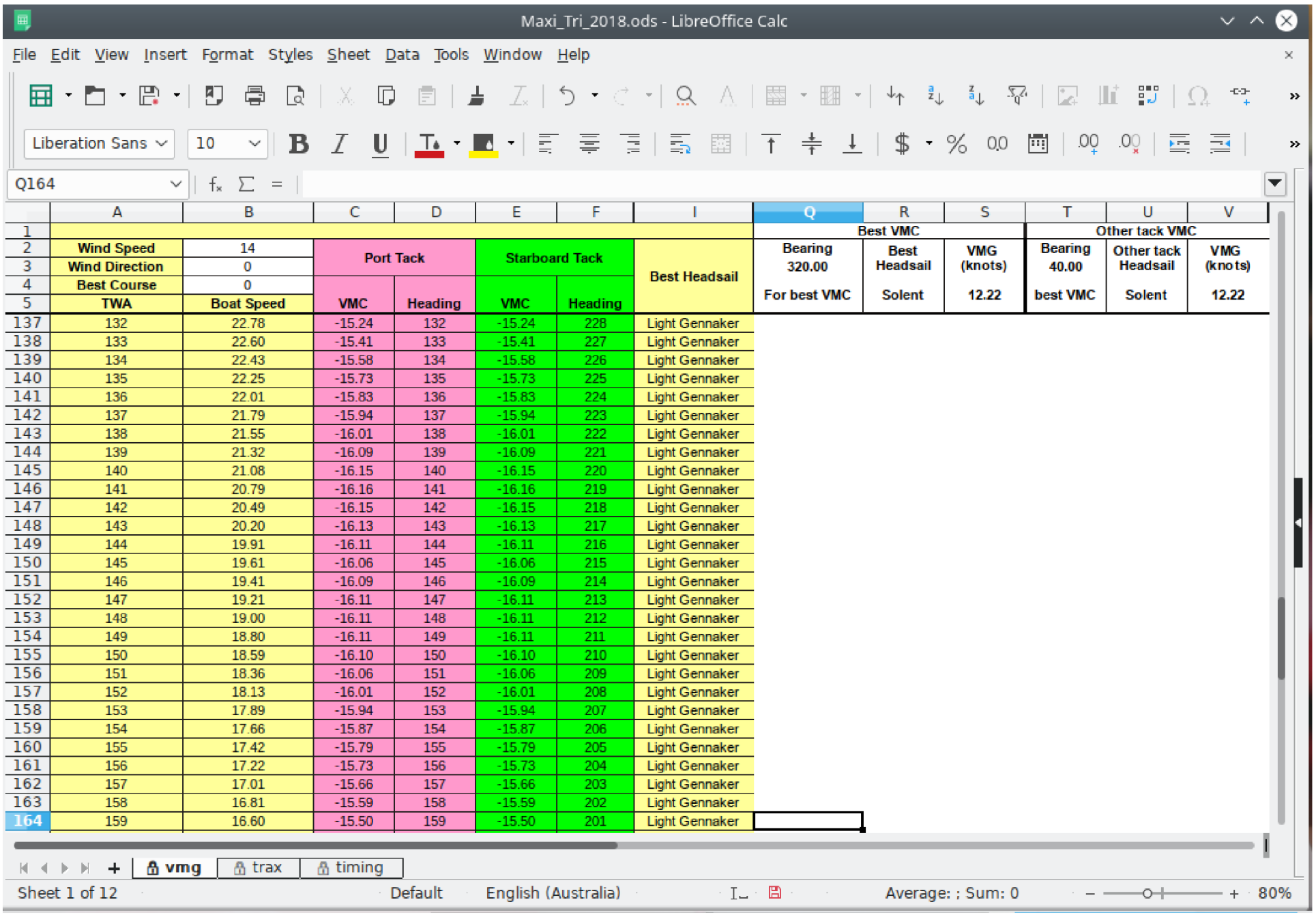
So in general, we hope for a change in wind direction that will hopefully send the next VMC heading back in the other direction. Therefore it is a strategy that pays when we are far away from the destination and there will be many weather updates before the finish. We must be careful also that the short term gain won't be penalised by sending us into a no wind patch at the next weather update.

Since for some reason maths seem to terrify people, I've made some calculators, using RG's polars, which calculate VMC (and VMG) for you automatically. I made them because I find them useful and I use them, and I don't mind sharing.

Now I'll explain how those calculators are used (well, how I use them myself). Personally, I find them easier to use than RG's polars.

4. VMC Calculators

All calculators look the same but each is dedicated to a set of polars. So pick the right one for the race. They can be found at:
https://madinstro.net/rg/



	A	B	C	D	E	F	I	Q	R	S	T	U	V
1								Best VMC			Other tack VMC		
2	Wind Speed	14	Port Tack		Starboard Tack		Best Headsail	Bearing 320.00	Best Headsail	VMG (knots)	Bearing 40.00	Other tack Headsail	VMG (knots)
3	Wind Direction	0											
4	Best Course	0											
5	TWA	Boat Speed	VMC	Heading	VMC	Heading		For best VMC	Solent	12.22	best VMC	Solent	12.22
137	132	22.78	-15.24	132	-15.24	228	Light Gennaker						
138	133	22.60	-15.41	133	-15.41	227	Light Gennaker						
139	134	22.43	-15.58	134	-15.58	226	Light Gennaker						
140	135	22.25	-15.73	135	-15.73	225	Light Gennaker						
141	136	22.01	-15.83	136	-15.83	224	Light Gennaker						
142	137	21.79	-15.94	137	-15.94	223	Light Gennaker						
143	138	21.55	-16.01	138	-16.01	222	Light Gennaker						
144	139	21.32	-16.09	139	-16.09	221	Light Gennaker						
145	140	21.08	-16.15	140	-16.15	220	Light Gennaker						
146	141	20.79	-16.16	141	-16.16	219	Light Gennaker						
147	142	20.49	-16.15	142	-16.15	218	Light Gennaker						
148	143	20.20	-16.13	143	-16.13	217	Light Gennaker						
149	144	19.91	-16.11	144	-16.11	216	Light Gennaker						
150	145	19.61	-16.06	145	-16.06	215	Light Gennaker						
151	146	19.41	-16.09	146	-16.09	214	Light Gennaker						
152	147	19.21	-16.11	147	-16.11	213	Light Gennaker						
153	148	19.00	-16.11	148	-16.11	212	Light Gennaker						
154	149	18.80	-16.11	149	-16.11	211	Light Gennaker						
155	150	18.59	-16.10	150	-16.10	210	Light Gennaker						
156	151	18.36	-16.06	151	-16.06	209	Light Gennaker						
157	152	18.13	-16.01	152	-16.01	208	Light Gennaker						
158	153	17.89	-15.94	153	-15.94	207	Light Gennaker						
159	154	17.66	-15.87	154	-15.87	206	Light Gennaker						
160	155	17.42	-15.79	155	-15.79	205	Light Gennaker						
161	156	17.22	-15.73	156	-15.73	204	Light Gennaker						
162	157	17.01	-15.66	157	-15.66	203	Light Gennaker						
163	158	16.81	-15.59	158	-15.59	202	Light Gennaker						
164	159	16.60	-15.50	159	-15.50	201	Light Gennaker						

In cell B2, enter the wind speed, don't forget to round up or down as RG does, the calculator works quite well with decimals but RG can't do it yet.

Here I entered 0 for the wind direction and 0 for the destination, this gives me directly the optimum TWA (40 and -40) to sail against the wind. They are the TWA in red (40) for a 14 knot wind for the Maxi trimaran 2018.

Column A gives the TWA from 1 to 180 degrees. Column B gives the boat speed. The green columns give the heading and VMC on starboard tack, and the red columns do likewise for port tack. Column I gives the best sail for the given TWA.

At the top, on the right, in white, the heading, sail and the best VMC, and next to it, the same on the opposite tack (please don't use that value as it is almost always the worst choice unless, as here, you are seeking the VMG not the VMC).

If instead of 0 I had used 180 for the direction I would have obtained the optimum TWAs for running before the wind.

In the general case here is what we get:

Maxi_Tri_2018.ods - LibreOffice Calc

File Edit View Insert Format Styles Sheet Data Tools Window Help

A135:AMJ135 =A134+1

	A	B	C	D	E	F	I	Q	R	S	T	U	V
1								Best VMC			Other tack VMC		
2	Wind Speed	14	Port Tack		Starboard Tack		Best Headsail	Bearing 101.00	Best Headsail	VMG (knots)	Bearing 151.00	Other tack Headsail	VMG (knots)
3	Wind Direction	331											
4	Best Course	120						For best VMC	Light Gennaker	21.87	best VMC	Light Gennaker	12.56
5	TWA	Boat Speed	VMC	Heading	VMC	Heading							
127	122	23.04	20.53	93	0.40	209	Reacher						
128	123	23.07	20.74	94	0.81	208	Reacher						
129	124	23.10	20.94	95	1.21	207	Reacher						
130	125	23.13	21.13	96	1.61	206	Reacher						
131	126	23.13	21.29	97	2.02	205	Light Gennaker						
132	127	23.13	21.45	98	2.42	204	Light Gennaker						
133	128	23.13	21.59	99	2.82	203	Light Gennaker						
134	129	23.13	21.74	100	3.22	202	Light Gennaker						
135	130	23.13	21.87	101	3.62	201	Light Gennaker						
136	131	22.96	21.84	102	3.99	200	Light Gennaker						
137	132	22.78	21.78	103	4.35	199	Light Gennaker						
138	133	22.60	21.72	104	4.70	198	Light Gennaker						
139	134	22.43	21.67	105	5.05	197	Light Gennaker						
140	135	22.25	21.59	106	5.38	196	Light Gennaker						
141	136	22.01	21.45	107	5.70	195	Light Gennaker						
142	137	21.79	21.31	108	6.01	194	Light Gennaker						
143	138	21.55	21.15	109	6.30	193	Light Gennaker						
144	139	21.32	21.00	110	6.59	192	Light Gennaker						
145	140	21.08	20.82	111	6.86	191	Light Gennaker						
146	141	20.79	20.59	112	7.11	190	Light Gennaker						
147	142	20.49	20.34	113	7.34	189	Light Gennaker						
148	143	20.20	20.09	114	7.57	188	Light Gennaker						
149	144	19.91	19.83	115	7.78	187	Light Gennaker						
150	145	19.61	19.56	116	7.98	186	Light Gennaker						
151	146	19.41	19.38	117	8.20	185	Light Gennaker						
152	147	19.21	19.20	118	8.42	184	Light Gennaker						
153	148	19.00	19.00	119	8.63	183	Light Gennaker						
154	149	18.80	18.80	120	8.83	182	Light Gennaker						

vmg trax timing

Sheet 1 of 12 · 1 rows, 1024 columns selected · Default · English (Australia) · Average: 80.1030166666667; Sum: 480.6181

So the best heading to reduce the distance in the direction 120 is 101, the sail to use is the light gennaker, the boat speed is 23.13 knots but the VMC is 21.87 knots. On the other hand, with the same TWA on starboard tack, the VMC is only 3.62 knots.

5. Useful tricks

5.1. If you wish to sail with the best VMG close hauled

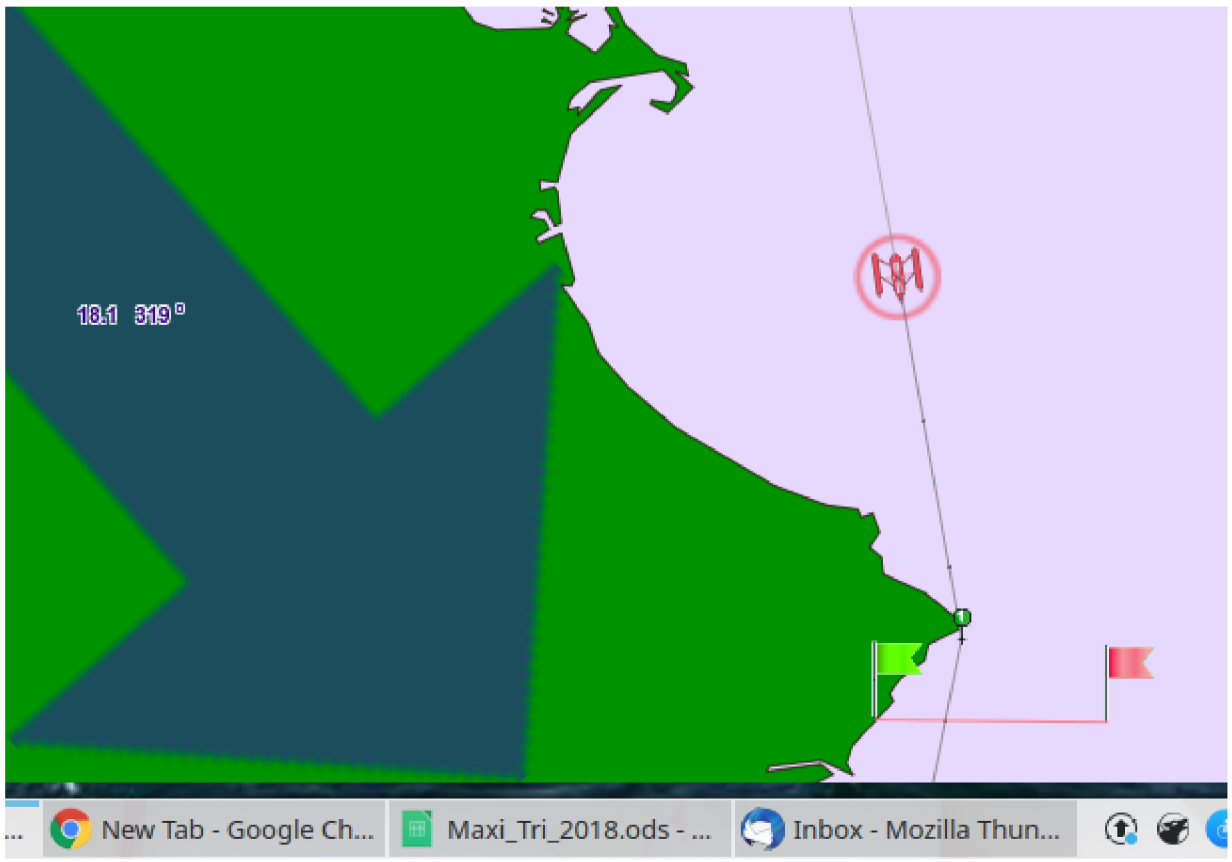
Place “=B3” in cell B4. The headings in cells Q3 and T3 will be those corresponding to the red TWA in RG’s polars for sailing against the wind. It’s handy when Programming the scheduler as while RG shows clearly the TWA, you need to add it to (or subtract it from) the wind direction to get the heading. Here it is already done automatically.

5.2. If you wish to sail with the best VMG running

Likewise place “=B3+180” in cell B4 and the headings appearing in Q3 and T3 will be those corresponding to RG’s red TWA for running downwind. They can be entered directly into the scheduler without having to add or subtract.

5.3. Best heading at the finish line

It’s also handy to find the best heading to cross the line.



Here Maxi trimaran 2018 is nearing the finish line with a heading of 172 to avoid grounding. But once past the cape, what heading (and what sail) will minimise the time to cross the line? In this case we need the perpendicular to the line. Here the line is horizontal (East – West) so the perpendicular is 180°.

With a wind of 18 knots, blowing from 319°, and with 180° as best course, I obtain 189° for the heading and Heavy Gennaker for the sail. These are the values I entered in the scheduler point 1.

6. Tab “trax”

The second tab of the calculator is "trax". It is used when the best course entered on the tab “vmg” must be reached. For instance there is a gap between two rocks that is where we want to go, the best course is the direction from the boat to the gap. Everything happens within one case and the wind will be the same all the way, in this case, unless the VMC is exactly the same as the best course (very unlikely), it is unlikely that it will be of any use to reach the gap.

That’s when 'trax' becomes handy. Trax calculates all the combinations of 2 headings to find the one that minimises the time to reach the destination. If we also give it the exact distance to the destination it will give the heading and lengths of those two legs as well as their duration (noting that there is still a little bug in RG’s game which makes the boats move only at about 95.4% of the indicated speed. Trax assumes there is no bug so all its times are 95.4% shorter than in the game).

For instance, with the Foil Max calculator: We wish to go in the 55°direction a distance of 72 nautical miles.

The wind is 37 knots coming from 330°.

‘VMG’ gives 75° as best VMC heading, but since we must really go to 55°...

	A	B	C	D	E	F	I	Q	R	S
1								Best VMC		
2	Wind Speed	37	Port Tack		Starboard Tack		Best Headsail	Bearing 75.00	Best Headsail	VMG (knots)
3	Wind Direction	230								
4	Best Course	55						For best VMC	Staysail	33.70
5	TWA	Boat Speed	VMC	Heading	VMC	Heading				
118	113	35.35	10.92	343	16.60	117	Staysail			
119	114	35.48	11.55	344	17.20	116	Staysail			
120	115	35.60	12.18	345	17.80	115	Staysail			
121	116	35.71	12.80	346	18.39	114	Staysail			
122	117	35.84	13.43	347	18.99	113	Staysail			
123	118	35.96	14.05	348	19.59	112	Staysail			
124	119	36.07	14.67	349	20.17	111	Staysail			

...we use tab ‘trax’:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Optimum Course	55	Degrees			Length	Bearing	Speed	Time			Total Time		
2	Optimum Distance	72	nm		1st Leg	29.66	29	34.88	00:51:01			02:12:10		
3					2nd leg	47.17	71	34.88	01:21:08					
4														
193														
194														
195														
196														

Which tells us that we need to travel 29.66 miles on a heading of 29°, and 47.17 miles on a heading of 71°. The order in which we execute those legs does not matter.

To know which sails to use we return to ‘vmg’ what is indicated for those two headings:

161	156	35.62	31.15	26	33.68	74	Staysail
162	157	35.37	31.23	27	33.64	73	Staysail
163	158	35.12	31.29	28	33.59	72	Staysail
164	159	34.88	31.35	29	33.53	71	Staysail
165	160	34.63	31.39	30	33.45	70	Staysail
166	161	34.22	31.26	31	33.20	69	Staysail

We see that 29 et 71 are both for a TWA of 159 and the sail is the “staysail” for both. Here they are the best VMG headings, but it happens that a different solution is found, sometimes the direct route wins in which case one of the legs has zero length, and sometime it’s two different headings.