Flow And Pressure Forces on a Thermowell

1. Compressive Stress

The pipe or vessel into which a thermowell is inserted is generally under pressure, compressive stresses are applied to the thermowell even when there is no flow. This pressure has a destabilising effect due to buckling and leads, or can lead, to a reduced allowable working stress. The pressure also has a lowering effect on the natural frequency of the thermowell, although this effect is rarely higher than 1 or 2%, but since it can be taken account of, we will account for it.

Since the effect is only approximate, we will base it on the buckling effect of a cantilevered column of uniform cross-section (ie we ignore the effect of the taper).

The compressive force, Fc, is $F_C = P \cdot \frac{\pi D_1^2}{4}$

And the corresponding compressive stress, fc, is $f_c = \frac{F_C}{A} = \frac{P D_1^2}{\left(D_1^2 - D_0^2\right)}$

The critical Euler stress, f_E (Buckling force) is $f_E = \frac{\pi^2 E I}{4 L^2 A}$, where E is the thermowell's Young's modulus and I is the second moment of area of the cross section at the support. Note that the formula strictly applies only to cylindrical thermowells, but it is thought to be approximately correct also for tapered thermowells because the load on the tip is less than F_E and it only approaches F_E as the sections get closer to the support.

$$I = \frac{\pi\left(D_1^4 - D_0^4\right)}{64}$$

$$A = \frac{\pi\left(D_1^2 - D_0^2\right)}{4}$$

hence $f_E = \frac{\pi^2 E\left(D_1^2 + D_0^2\right)}{64 L^2}$, Euler's critical stress. As a practical value, the maximum working stress should not be greater than either $f_E/2$ or the allowable stress, whichever is less.

2. Effect of stress on natural frequency

Compressive stress has for effect to lower the resonance frequency of a structural member. When the stress reaches the Euler critical stress, the resonance frequency becomes zero. The converse is also true, tension increases the resonance frequency. The relation between frequency and compressive stress is as follows:

$$\frac{\omega_n}{\omega_{n_o}}=1-\left(\frac{f_C}{f_E}\right)^2$$

This formula is easily proven for a column simply supported at both ends. It is also approximately true in more general structural cases (ref 1) and is the basis for the experimental in-situ determination of critical loads without destruction of the structures being tested. Two frequency measurements, one with the structure unloaded and the second under a reasonably safe load are made, this allows extrapolation of this load to the critical load. Conversely, knowing the ratio of the compressive stress to Euler's critical stress, we can easily infer the effective natural pulsation of the thermowell knowing its natural pulsation when not compressed.

3. Forces due to flow

Two forces are applied to the thermowell by the flow past it. One is in the direction of the flow and is called the drag pressure. This pressure is given, per unit of projected area in the direction of the flow, as:

$P_D = C_D \frac{\rho V^2}{2}$, with Cd = 1.2 for cylinders in turbulent flow. (Reference 2)

The second pressure is oscillatory in nature and is called the Von Karman pressure. It is applied at right angle to the flow and is:

$P_K = C_K \frac{\rho V^2}{2}$, with Ck = 1.7 for the Von Karman coefficient for cylinders. (Reference 3).

If the bending forces were static, they would combine according to Pythagoras's rule for right angled triangles, ie : $P_T=\sqrt{P_D^2+P_K^2} = \rho\frac{V^2}{2}\sqrt{C_K^2+C_D^2} = 1.75\rho\frac{V^2}{2}$

The bending moment at the support is:

$$Mt=P_T\left(\frac{D_1+2D_2}{6}\right)L^2$$

Where D_1 is the root diameter, D_2 is the tip diameter and L is the thermowell length.

Hence the "static" bending stress is:

$$f_b=\frac{Mt}{Z}=\frac{Mt}{I}\cdot\frac{D_1}{2}$$

$$f_b = \frac{32\,Mt}{\pi D_1^3 \left(1 - \frac{D_0^4}{D_1^4}\right)} = 1.75 \rho \frac{V^2}{2} L^2 \left(\frac{D_1 + 2D_2}{6}\right) \frac{32}{\pi D_1^3 \left(1 - \frac{D_0^4}{D_1^4}\right)}$$

$$f_b = 1.75 \frac{8\rho (D_1 + 2D_2) L^2 V^2}{3\pi D_1^3 \left(1 - \frac{D_0^4}{D_1^4}\right)}$$

4. Dynamic effect – Moment Multiplier

In a second order, non dissipative system (ie a mass/spring system), which is subjected to a harmonic force F=Fo.cos(ωt), the dynamic maximum displacement is different, and larger, than the displacement obtained under a static Fo force for all frequency values below ω_n.

If K is the spring coefficient, and m is the mass, then the natural pulsation is

$$\omega_n = \sqrt{\frac{K}{m}}$$

The "static" deflection of the spring, z_o, is $z_o = \frac{F_o}{K}$ when ω=0.

But when ω increases and approaches ω_n, the maximum deflection, z, increases, becoming infinite when $\omega = \omega_n$.

The relation for a second order system when $\omega < \omega_n$ is:

$$z = z_o \cdot \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

The term $K_m = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$ is called the stress (or strain) multiplier. Note that ASME PTC 19.3 calls moment multiplier Fm, Fm = Km – 1. It then multiplies the static stress by (1+Fm) and therefore obtains the same result. I can't see any good reason for doing it their way when the above formula is actually simpler.

We assume that what applies to a second order system also applies approximately to a distributed system before the 1st critical frequency is reached. Therefore the total stress is taken as:

$$f = K_m (f_b + f_C)$$

(Note: both fb and fc are counted positive in the formula above, just because fatigue is usually a tensile failure does not mean that compressive stresses will reduce the incidence of failure, they will actually increase it as they lead to a decrease in stability)

5. Acceptance of Thermowell

The thermowell is acceptable if:

- The wake frequency is less than 75% of the natural frequency of the thermowell in compressed mode. (Note PTC 19.3 assumes 80%, but they underestimate the natural frequency making their results more conservative anyway).

- The total stress, f, after allowance for stress multiplier, is less than the allowable stress for the material. (Check AS1210 for the correct values as it varies with temperature and materials). Note that PTC 19.3 calculates a "Maximum Length" by what I consider to be a seriously flawed method giving the illusion of a greater safety factor than actually exists (see note below).

- The total stress, f, after allowance for stress multiplier, is also less than 50% of Euler's buckling stress.

- Finally, and optionally, a fatigue analysis should be carried out such that based on the effective loading (f/f_{yield}), the maximum number of permissible cycles can be calculated. Knowing the shedding frequency, this will yield an estimate of the useful life of the thermowell. After this period, it should be replaced as it is under danger of fatigue failure. This step is important if f is greater than 20% of the yield stress, or for when the material is prone to fatigue failures.

Note about Maximum Length: PTC 19.3 calculates a "Maximum Length" which supposedly the thermowell is allowed to reach before danger is reached. This is grossly misleading as this "Maximum Length" is calculated on the basis of a Stress Multiplier Km=(1 +Fm) which is based on the actual length of the thermowell. If the thermowell length is made longer, all other parameters remaining the same, then its natural frequency will decrease, bringing it closer to the shedding frequency. This in turn will cause the Stress Multiplier to increase, leading in turn to a lesser "Maximum Length".

The concept of maximum length is seriously flawed and should not be used. The concept of allowable stress, taking fatigue into account, is the only viable alternative.

Finding suitable data for fatigue design is difficult. AS1210 supplement 1 - 1990 (Reference 4) deals with fatigue but stops their stress/cycle curves at 10^6 cycles. For a thermowell vibrating at 100 Hz, this leads to a maximum life of 10000 seconds, just short of 3 hrs. It is known that for many materials, steel and its alloys for instance, there exists an asymptotic stress below which the number of cycles becomes infinite. Publication of such data would be eminently useful to the design of thermowells.

On a positive side, most books on fatigue say that for steels, the failure stress doesn't go much below the stress level at 10^6 cycles. However, this is not the case for other metals.

6. References

(a) Résistance des Matériaux. Tome II. Charles Massonet. Dunod
(b) Perry Chemical Engineers Handbook. 7th Edition. Figure 6-57
(c) Perry Chemical Engineers Handbook. 7th Edition. Equation 6-196
(d) Australian Standard AS1210 - Unfired Pressure Vessels, Supplement 1, Appendix SC. (1990)